

Revealed Preference in Bidding: Empirical Evidence from Recent Spectrum Auctions

Lawrence Ausubel

University of Maryland

Oleg Baranov

University of Colorado

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Activity Rules

At a glance:

- Activity rules in dynamic auctions are intended to prevent “bid sniping”: a bidder **must** submit bids in early rounds in order to be allowed to continue to submit bids in later rounds
- Spectrum auctions have traditionally required monotonicity in eligibility points
- Various researchers have suggested activity rules based on revealed preference
- In particular, Ausubel and Baranov (2014) have advocated activity rules based on Generalized Axiom of Revealed Preference (GARP)

Revealed Preference for quasilinear preferences

Given a clock bidding history $(p_1, x_1), \dots, (p_t, x_t)$

Weak Axiom of Revealed Preference (WARP):

$$p_s(x_s - x_z) + p_z(x_z - x_s) \leq 0 \quad \forall s, z \in \{1, \dots, t\}$$

Generalized Axiom of Revealed Preference (GARP):

$$p_{k_1}(x_{k_1} - x_{k_m}) + p_{k_2}(x_{k_2} - x_{k_1}) + \dots + p_{k_m}(x_{k_m} - x_{k_{m-1}}) \leq 0 \\ \forall k_1, \dots, k_m \in \{1, \dots, t\}$$

Afriat's Theorem (1967):

A finite set of data is consistent with utility maximization if and only if it satisfies GARP.

Intuition for the Revealed Preference Activity Rule:

As prices rise, bidders cannot bid for packages that became relatively more expensive.

Combinatorial Clock Auction – CCA 101

At a glance:

A three-phase auction with package bidding, consisting of a clock auction phase, followed by a supplementary round, and concluding with an assignment stage

- **Clock Auction Phase:** Auctioneer announces prices and bidders respond with quantities; bidders are constrained by the clock activity rule
- **Supplementary Round:** Bidders can supplement their bids from the clock rounds with additional package bids which are constrained by the activity rule
- **Assignment Stage:** Sealed-bid auction to map generic spectrum to specific frequencies
- Allocation is determined by finding a feasible allocation that maximizes total bid value (*Winner Determination Problem*)
- Payments are determined by a *core-selecting mechanism*

CCAs to date:

Country and Auction	Year	Revenue
Trinidad and Tobago	2005	\$ 25.1 million (\$US)
UK 10-40 GHz	2008	£ 1.43 million
UK L-Band	2008	£ 8.33 million
Netherlands 2.6 GHz	2010	€ 2.63 million
Denmark 2.6 GHz	2010	DKK 1.01 billion
Switzerland Multi-Band	2012	CHF 996 million
Denmark 800 MHz	2012	DKK 739 million
Ireland Multi-Band	2012	€ 482 million
Netherlands Multi-Band	2012	€ 3.8 billion
UK 4G	2013	£ 2.34 billion
Australia Digital Dividend	2013	\$ 1.96 billion (\$AU)
Austria Multi-Band	2013	€ 2.01 billion
Slovakia 800, 1800 and 2600 MHz	2013	€ 164 million
Canada 700 MHz	2014	\$ 5.27 billion (\$CA)
Slovenia Multi-Band	2014	€ 149 million

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UK 4G Spectrum Auction (2013): Some Details

Setup:

- Lots:** Supply of 4 A1 lots, one A2 lot, 14 C lots and 9 E lots
 some of C lots might have been won as D1 or D2 lots
- Bandplan:** Vodafone, Telefonica, EE, Three, Niche(BT), HKT and MLL
- Bidders:** provision (i.e. competition policy) that there should be at least 4 credible national competitors after the auction
- Spectrum Floor:**
- Activity Rules:** *Clock Stage: Pure eligibility-point monotonicity*
Supplementary Stage: Relative Cap
- Results:** all lots were sold; £ 2.34 billion raised

Bidder	Vodafone	Telefonica	EE	Three	Niche
Package	2-A1, 4-C, 5-E	1-A2	1-A1, 7-C	1-A1	3-C, 4-E
Price	£ 803 mil	£ 550 mil	£ 589 mil	£ 225 mil	£ 202 mil

Canadian 700 MHz Auction (2014): Some Details

Setup:

Regions:	Tier-2 Partition: 14 Service Areas
Lots:	5 paired lots (A, B/C and C1/C2) and 2 unpaired lots (D/E) in each service area
Bidders:	10 bidders
Activity Rules:	<i>Clock Stage: Hybrid activity rule</i> <i>Supplementary Stage: Intermediate Cap + Final Cap</i>
Results:	97 out of 98 licenses sold; 5.27 billion (CAD) raised

Eligibility-Preserving Bids

Background:

- most common bidding inconsistency
- “almost” no consequences for bidders
- can be used to achieve other objectives: manipulate clock price trajectory

Example: Vodafone in UK 4G Auction

Round	Bid - (A1,A2,C,E)	Price (A1, A2)	Difference
19	(0, 1, 8, 7)	(£316 mil, £608 mil)	£24 mil
20	(2, 0, 8, 7)	(£332 mil, £638 mil)	£26 mil
21	(0, 1, 8, 7)	(£349 mil, £638 mil)	£60 mil
22	(2, 0, 8, 7)	(£366 mil, £670 mil)	£62 mil
23	(2, 0, 8, 7)	(£384 mil, £670 mil)	£98 mil
24	(2, 0, 8, 7)	(£403 mil, £670 mil)	£136 mil

Eligibility-Reducing Bids

Background:

- play a very important role under the monotonic activity rule
- even more important role under the hybrid activity rule
- can be strategically used to increase supplementary bid limits or to place relaxed bids

Example: Bragg in Canadian 700 MHz Auction

Round	Package						Activity
	NL	NS	NB	NQC	NON	AB	
36	C1/C2 +D/E	C1/C2 +D/E	C1/C2 +D/E		B/C + C1/C2 +D/E		109
37				A		A	93
38	C1/C2 +D/E	C1/C2 +D/E	C1/C2 +D/E		C1/C2 +D/E		93

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Example: Bragg in Canadian 700 MHz Auction

Round	Package						Activity
	NL	NS	NB	NQC	NON	AB	
67	B/C +D/E	B/C +D/E	B/C +D/E		B/C +D/E		93
68	A	A	A		A		62
69	B/C	B/C	B/C		B/C		62

Relaxed Bids

Background:

- possible under the Hybrid Activity Rule
- a relaxed bid is very likely to create inconsistency since only very limited set of revealed-preference constraints is tested by the Hybrid Activity Rule

Example: Bragg in the Canadian 700 MHz Auction

Round	Package						Activity
	NL	NS	NB	NON	MB	SK	
78	C1/C2	C1/C2	C1/C2	C1/C2			62
...
93	C1/C2	C1/C2	C1/C2	C1/C2			62
94	C1/C2	C1/C2	C1/C2	C1/C2	B/C	B/C	106
95	C1/C2	C1/C2	C1/C2	C1/C2			62

Forced Bids

Background:

- strict monotonic activity rule can sometimes preclude bidders from bidding truthfully
- it can happen for any choice of eligibility points

Example: EE in UK 4G Auction

Round	Bid - (A1,A2,C,E)	Price (C, E)	Difference
37	(0, 0, 8, 0)	(£87.6 mil, £1.194 mil)	£690.054 mil
38	(0, 0, 0, 9)	(£92 mil, £1.43 mil)	£723.13 mil
...
45	(0, 0, 0, 9)	(£92 mil, £5.11 mil)	£690.01 mil
46	(0, 0, 0, 9)	(£92 mil, £6.4 mil)	£678.4 mil
...
52	(0, 0, 0, 9)	(£92 mil, £24.4 mil)	£516.4 mil

Activity Rules - Supplementary Round

Definition: Revealed Preference Constraint

A revealed preference constraint for package x with respect to the clock round t is

$$b(x) \leq b(x_t) + p_t(x - x_t)$$

where $b(x)$ is the bid amount for package x , x_t is the package demanded in Round t , $b(x_t)$ is the bid amount for package x_t , and p_t are the clock prices in Round t .

Popular Choices:

- **Relative Cap** – A bid for the package x should satisfy the revealed preference constraint with respect to the last clock round in which bidder's eligibility was greater than or equal to the total points associated with the package x .
- **Final Cap** – A bid for the package x should satisfy the revealed preference constraint with respect to the final clock round.

Activity Rules - Supplementary Round

Definition: Revealed Preference Constraint

A revealed preference constraint for package x with respect to the clock round t is

$$b(x) \leq b(x_t) + p_t(x - x_t)$$

where $b(x)$ is the bid amount for package x , x_t is the package demanded in Round t , $b(x_t)$ is the bid amount for package x_t , and p_t are the clock prices in Round t .

Popular Choices:

- **Intermediate Cap** – A bid for the package x should satisfy the revealed preference constraint with respect to all eligibility-reducing rounds starting from the last clock round in which the bidders eligibility was greater than or equal to the total points associated with the package x .

Net Incremental Exposure

Background:

- incremental value (on top of the final clock prices) of bidder's opponents for its final clock package
- net the "knockout bid" amount of $(p_T - r)U$
- netted amounts: £ 743.2 million for UK, 288.442 million CAD for Canada

Net Incremental Exposure in UK 4G Auction

Bidder	Relative Cap	Intermediate Cap	Relative Cap + Final Cap	Intermediate Cap + Final Cap
Vodafone	+885 (79%)	+206 (18%)	0	0
Telefonica	+885 (99%)	+206 (23%)	0	0
EE	+568 (258%)	0	0	0
Niche	+1083 (588%)	+206 (112%)	0	0

Net Incremental Exposure

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- incremental value (on top of the final clock prices) of bidder's opponents for its final clock package
- net the "knockout bid" amount of $(p_T - r)U$
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Net Incremental Exposure in Canadian 700 MHz Auction

Bidder	Relative Cap	Intermediate Cap	Relative Cap + Final Cap	Intermediate Cap + Final Cap
Rogers	+1515 (38%)	+217 (5%)	-75(-2%)	-105(-3%)
Bell	+1711 (125%)	+1268 (94%)	-75(-6%)	-105(-8%)
TELUS	+1749 (133%)	+1402 (106%)	-75(-6%)	-105(-8%)
Videotron	+1925 (411%)	+1576 (336%)	-75(-16%)	-105(-22%)
Bragg	+1810 ($\approx 10^3\%$)	+1574 ($\approx 10^3\%$)	0	0
SaskTel	+2126 ($\approx 10^4\%$)	+1741 ($\approx 10^4\%$)	0	0
MTS	+2156 ($\approx 10^4\%$)	+1755 ($\approx 10^4\%$)	0	0
Feenix	+2165 ($\approx 10^5\%$)	+1746 ($\approx 10^5\%$)	0	0

Extra Analysis for UK

Simulating Clock Rounds using reported values:

- Vodafone and Niche were broadly consistent (slight delays in dropping lots)
- Three, Telefonica and EE exhibited larger delays in dropping lots

Summary for UK

- Telefonica, HKT and MLL were fully consistent with GARP
- With deletion of round 41, Niche was consistent with GARP
- With deletion of rounds 20 and 22-24, Vodafone was consistent with GARP
- With deletion of rounds 16, 24 and 25, Three was consistent with GARP
- EE's near-end bidding exhibited two inconsistencies:
 - In round 37, EE would have been expected to revert from 8-C to 2-A1 and 9E (but couldn't do this, the shift requires eligibility points to go from 1200 to 4509!)
 - In round 50, EE would have been expected to revert from 9-E to 8-C (but couldn't do this, the shift requires eligibility points to go from 9 to 1200!)

Summary:

Problems with the point-based activity rule:

- bidders are not incentivized to pay attention to relative clock prices in the auction until the very end
- eligibility-preserving rounds are meaningless for the price-discovery process (bids are not credible)
- eligibility-reducing rounds can be abused
- nonuniform imposition of the revealed-preference constraints results in artificial bid sniping opportunities (Final Cap)

Solution:

- it is critical that bidders utilize the clock prices to resolve demand overlap
- activity rule should force bidders to react to clock price trajectory

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Problems with the point-based activity rule:

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Solution:

- it is critical that bidders utilize the clock prices to resolve demand overlap
- activity rule should force bidders to react to clock price trajectory

GARP-based Activity Rule

GARP Activity Rule

Definition: GARP Clock Activity Rule

Given a prior clock bidding history $(p_1, x_1), \dots, (p_{t-1}, x_{t-1})$, a package x_t is feasible in Round t with prices p_t if there exists a set of numbers $\{v_i\}_{i=1}^t$ such that:

$$v_i \leq v_j + p_j(x_i - x_j) \quad \forall i, j \in \{1, \dots, t\}$$

Advantageous:

- the strictest rule that permits straightforward bidding
- satisfies the “no-dead-end” property

Practical problems:

- GARP does not limit set of feasible packages from below
- bidding budgets
- accidental suboptimal choices (bidding mistakes) and common value uncertainty

GARP Refinement - SARP Activity Rule

Example: Niche (BT) in UK 4G Auction

Round	Bid - (C,E)	Price (C, E)
40	(3, 5)	(£92 mil, £2.06 mil)
42	(2, 5)	(£92 mil, £2.96 mil)

Discussion:

- By bidding (2,5) in R42, Niche have demonstrated indifference between (3, 5) and (2, 5) in R40 and R42

Definition: SARP Clock Activity Rule

Given a prior clock bidding history $(p_1, x_1), \dots, (p_{t-1}, x_{t-1})$, a package x_t is feasible in Round t with prices p_t if there exists a set of numbers $\{v_i\}_{i=1}^t$ such that:

$$\begin{aligned}
 v_i &= v_j & \forall i, j \in \{1, \dots, t\} : x_i &= x_j \\
 v_i &< v_j + p_j(x_i - x_j) & \forall i, j \in \{1, \dots, t\} : x_i &\neq x_j
 \end{aligned}$$

GARP Relaxation - Budgets

Definition: Revealed Budget

Given a clock bidding history $(p_1, x_1), \dots, (p_t, x_t)$, the revealed budget B^* is defined as the solution to the following optimization problem: ($B^* = +\infty$ if unconstrained)

$$B^* = \max B$$

subject to:

$$v_i \leq v_j + p_j(x_i - x_j) \quad \forall i, j \in \{1, \dots, t\} : p_j x_i \leq B$$

Definition: Budget-Constrained GARP Clock Rule

A package x_t is feasible in Round t with prices p_t if

$$p_j x_j \leq B^* \quad \forall j \in \{1, \dots, t\}$$

GARP Relaxation - General

Mistakes and common uncertainties

- bidder has to be “optimal” in every round
- activity rule can provide bidders some room for suboptimal choices

Definition: Relaxed GARP Clock Rule

Given a prior clock bidding history $(p_1, x_1), \dots, (p_{t-1}, x_{t-1})$, a package x_t is feasible in Round t with prices p_t if there exists a set of numbers $\{v_i\}_{i=1}^t$ and a set of numbers $\{\lambda_{ij}\}_{i,j=1}^t \geq 0$ such that:

$$v_i \leq v_j + p_j(x_i - x_j) + \lambda_{ij} \quad \forall i, j \in \{1, \dots, t\}$$

Example: Relaxed GARP Clock Rule

- $\lambda_{ij} = 5$ million
- The bidder can select any suboptimal package in a 5 million radius from the optimal package

Concluding Remarks:

Two attractive features of GARP-based activity rules:

- 1 No need to assign eligibility points
 - nontransparent and time-consuming
 - heavy lobbying by stakeholders
 - eligibility points frequently duplicate reserve prices (redundancy)
 - possibility of mistakes (UK 4G Auction)
- 2 Nonmonotonic price trajectory
 - eligibility-point rules requires monotonic price trajectory
 - GARP can work with nonmonotonic price trajectories (classical tatonnement)